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Form factors of $D_s^+ \to \phi \bar{\ell} \nu$ decay in QCD light cone sum rule

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Abstract. We calculate the form factors V, A_1, A_2 and A_0 appearing in the $D_s \to \phi$ transition by the light cone QCD sum rule method. We compare our results on these form factors with the current experimental results and existing theoretical calculations.

1 Introduction

Semileptonic decays of mesons containing charm and beauty quarks constitute a very important class of decays for studying the strong and weak interactions. These decay modes of heavy flavored mesons are much clearer samples than those of the hadronic decay modes, since leptons do not participate in the strong interaction.

Therefore, the study of these decays is one efficient way for determining the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, as well as for understanding the origin of CP violation which is related to the structure of the CKM matrix in the standard model (SM).

An accurate determination of CKM matrix elements, obviously, depends crucially on the possibility of controlling the effects of the strong interactions. For exclusive decays, where the initial and final states of the hadrons are known, the main job is to calculate various transition form factors, which involve all the long distance QCD dynamics. So, some non-perturbative approach for estimating the long distance effects is needed. Several methods have been used to treat these effects, such as the quark model, QCD sum rules, lattice theory, chiral perturbation theory, etc. Among these approaches, the QCD sum rule method occupies a special place, since it is based on the very first principles of QCD.

The method of QCD sum rules [1] has been successfully applied to wide variety of problems of hadron physics (see [2,3] and references therein). In this method, physical observables of hadrons are related to the QCD vacuum via a few condensates. The semileptonic decay $D \to \bar{K}^0 e \bar{\nu}_e$ was firstly studied by QCD sum rules with the 3-point correlation function in [4]. This method, then, is successfully extended to the study of other semileptonic decay decays of the D and B mesons, i.e., $D^+ \to \bar{K}^0 e^+ \nu_e$, $D^+ \to \bar{K}^{0*} e^+ \nu_e$ [5], $D \to \pi e \bar{\nu}_e$, $D \to \rho e \bar{\nu}_e$ [6], $B \to D(D^*) \ell \bar{\nu}_\ell$ [7] and $B \to \pi \ell \bar{\nu}_\ell$ [8].

However, this method inherits some problems, the main one being that some of the form factors have a nasty behavior in the heavy quark limit, $m_Q \to \infty$. In order to overcome the problems of the traditional QCD sum rules, an alternative method, namely the light cone QCD sum rule method (LCQSR), was developed in [9] and is regarded as an efficient tool in studying exclusive processes which involve the emission of a light particle.

The LCQSR is based on the operator product expansion (OPE) near the light cone $x^2 \approx 0$, which is an expansion over the twist of the operators, rather than the dimensions as in the traditional QCD sum rules. All non-perturbative dynamics is parameterized by the so-called light cone wave functions, instead of the vacuum condensates in the traditional sum rules, which represents the matrix elements of the non-local operators between the vacuum and the corresponding particle (more about this method can be found in [3,10])

The LCQSR has a wide range of applications to numerous problems of hadron physics. One of the promising ways for obtaining information about CKM matrix elements, as well as about wave functions, is studying the semileptonic decays.

In this work we study $D_s^+ \to \phi \bar{\ell} \nu$ decay in LCQSR. This decay mode has been measured in experiments in [11–14]. Note that the $D_s \to \phi$ transition form factors are calculated in the framework of traditional 3-point QCD sum rules in [15,16], but the results do not confirm each other. Therefore we decided to calculate $D_s \to \phi$ form factors using light cone sum rules as an alternative approach to the traditional sum rules.

This paper is organized as follows. In Sect. 2 we derive the sum rules for the transition form factor. Section 3 is devoted to the numerical analysis and discussions and contains a summary of our results and the conclusions.

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2 Light cone sum rules for the $D_s \to \phi$ transition form factors

The weak transition matrix element $D_s \to \phi$ can be parametrized in terms of the form factors in the following way:

$$\begin{aligned}
&\langle\phi(P)|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s(p_{D_s})\rangle \\
&= -\mathrm{i}\varepsilon^*_\mu(m_{D_s}+m_\phi)A_1(q^2) \\
&\quad +\mathrm{i}\,(p_{D_s}+P)_\mu\,(\varepsilon^* q)\,\frac{A_2(q^2)}{m_{D_s}+m_\phi} \\
&\quad +\mathrm{i}q_\mu\,(\varepsilon^* q)\,\frac{2m_\phi}{q^2}\left[A_3(q^2)-A_0(q^2)\right] \\
&\quad +\frac{2V(q^2)}{m_{D_s}+m_\phi}\epsilon_{\mu\alpha\beta\gamma}\varepsilon^{*\alpha}q^\beta P^\gamma\,, \qquad (1)
\end{aligned}$$

where $q = p_{D_s} - P$ is the momentum transfer, P and ε are the momentum and polarization four–vectors of the ϕ meson, respectively, and p_{D_s} is the four momentum of the D_s meson.

In this section we derive sum rules for these form factors. In order to calculate the form factors of the semileptonic $D_s \to \phi\ell\nu$ decay, we consider the following correlator function:

$$\begin{aligned}
&\Pi_\mu(P,q) = \mathrm{i}\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}qx} \\
&\quad\times\langle\phi(P)T\left[\bar{s}(x)\gamma_\mu(1-\gamma_5)c(x)\bar{c}(0)(1-\gamma_5)s(0)\right]|0\rangle \\
&= \Gamma^0\varepsilon^*_\mu - \Gamma^+\frac{\varepsilon^* q}{Pq}(2P+q)_\mu - \Gamma^-\frac{\varepsilon^* q}{Pq}q_\mu \\
&\quad +\mathrm{i}\Gamma^V\varepsilon_{\mu\alpha\beta\gamma}\varepsilon^{*\alpha}q^\beta P^\gamma\,. \qquad (2)
\end{aligned}$$

The Lorentz invariant functions $\Gamma^{0,\pm,V}$ can be calculated in QCD for large Euclidean $p^2_{D_s}$, or, to put it more correctly, when $m_c^2 - p^2_{D_s} \ll 0$, the correlation function (1) is dominated by the region of small x^2 and can be systematically expanded in powers of the deviation from the light cone $x^2 = 0$.

The main reason for choosing the chiral current $\bar{c}(1-\gamma_5)s$ is that in this case many of the twist-3 wave functions, which are poorly known and cause the main uncertainties to the sum rules, can effectively be eliminated and provide results with less uncertainties. The chiral current approach has been applied to studying the $B \to \pi$ [17,18] and $B \to \eta$ [19] weak form factors.

Let us discuss firstly the hadronic representation of the correlator. This can be done by inserting the complete set of intermediate states with the same quantum numbers as the current operator $\bar{c}(1-\gamma_5)s$ in the correlation function. By isolating the pole term of the lowest pseudoscalar D_s meson, we get the following representation of the correlator function from the hadron side:

$$\begin{aligned}
&\Pi_\mu(P,q) \\
&= \frac{\langle\phi|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s\rangle\langle D_s|\bar{c}(1-\gamma_5)s|0\rangle}{m^2_{D_s}-(P+q)^2} \\
&\quad +\sum_h\frac{\langle\phi|\bar{s}\gamma_\mu(1-\gamma_5)c|h\rangle\langle h|\bar{c}(1-\gamma_5)s|0\rangle}{m_h^2-(P+q)^2}\,. \qquad (3)
\end{aligned}$$

Here, we would like to make the following remark. Due to the chiral structure of the $\bar{c}(1-\gamma_5)s$ current, the correlation function receives a contribution from the scalar $J^P = 0^+$ D_s mesons, in addition to the pseudoscalar D_s mesons. Recent BaBar, BELLE and CLEO data indicate that the lowest 0^+ D_s meson mass is 2.317 GeV [20]. Therefore, the lowest pseudoscalar D_s meson, as well as the 0^+ D_s mesons, can contribute to the dispersion integral. We can avoid the pollution from the scalar resonances by choosing the continuum threshold s_0 slightly below the squared mass of the lowest scalar D_s meson.

For the invariant amplitudes $\Gamma^{0,\pm,V}$, one can write a general dispersion relation in the $p^2_{D_s} = (P+q)^2$ variable

$$\Gamma^i\left(q^2,(P+q)^2\right) = \int \mathrm{d}s\frac{\rho^i(s)}{s-(P+q)^2} + \text{subtr.}\,,$$

where the spectral densities corresponding to (2) can easily be calculated. As an illustration of this fact, we present the result for Γ^0:

$$\begin{aligned}
&\rho^{(0)}(s) \qquad (4) \\
&= \frac{f_{D_s}m^2_{D_s}}{m_c+m_s}(m_{D_s}+m_V)A_1(q^2)\delta\left(s-m^2_{D_s}\right) + \rho^{(0)h}(s)\,.
\end{aligned}$$

The first term in (4) represents the contribution of the ground state D_s meson. In deriving (2), we have used

$$\langle D_s|\bar{c}(1-\gamma_5)s|0\rangle = \mathrm{i}\frac{f_{D_s}m^2_{D_s}}{m_c+m_s}\,.$$

The second term in (4) corresponds to the spectral density of the higher resonances and continuum. The spectral density $\rho^{(0)h}(s)$ can be approximated by invoking the quark hadron duality ansatz

$$\rho^{(0)h}(s) = \rho^{(0)\mathrm{QCD}}(s-s_0)\,.$$

So for the hadronic representation of the invariant amplitude $\Gamma^{(0)}$ we have

$$\begin{aligned}
\Gamma^{(0)} &= \frac{f_{D_s}m^2_{D_s}}{m_c+m_s}\frac{m_{D_s}+m_\phi}{m^2_{D_s}-(P+q)^2}A_1(q^2) \\
&\quad + \int_{s_0}^\infty \mathrm{d}s\frac{\rho^{(0)\mathrm{QCD}}(s)}{s-(P+q)^2} + \text{subtr.} \qquad (5)
\end{aligned}$$

Hadronic representations for the other invariant amplitudes can be constructed in precisely the same manner.

In order to obtain sum rules for the form factors A_1, A_2, A_0 and V, we must calculate the correlator from the QCD side. This calculation can be performed by using the light cone OPE. The contributions to OPE can be obtained by contracting the quark fields to a full c-quark propagator, i.e.,

$$\begin{aligned}
&\Pi_\mu(P,q) \qquad (6) \\
&= \mathrm{i}\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}qx}\langle\phi|\bar{s}\gamma_\mu(1-\gamma_5)S_c(x)(1-\gamma_5)s(0)|0\rangle
\end{aligned}$$

$$= \frac{\mathrm{i}}{4}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx}\left[\mathrm{Tr}\,\gamma_\mu(1-\gamma_5)S_c(x)(1-\gamma_5)\Gamma_i\right] \times\langle\phi|\bar{s}\Gamma^i s|0\rangle\,,$$

where Γ^i is the full set of the Dirac matrices $\Gamma^i = (I,\ \gamma_5,\ \gamma_\alpha,\ i\gamma_\alpha\gamma_5,\ \sigma_{\alpha\beta}/\sqrt{2})$, and

$$\begin{aligned}\mathrm{i}S_c(x) &= \mathrm{i}S_c^{(0)}(x)\\ &-\mathrm{i}g_\mathrm{s}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\int_0^1\mathrm{d}u\\ &\times\left[\frac{1}{2}\frac{\not{k}+m_c}{(m_c^2-k^2)^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}\right.\\ &\left.+\frac{1}{m_c^2-k^2}ux_\mu G^{\mu\nu}(ux)\gamma_\nu\right].\end{aligned} \tag{7}$$

Here, $G_{\mu\nu}$ is the gluonic field strength, g_s is the strong coupling constant and $S_c^{(0)}$ represents a free c-quark propagator

$$S_c^{(0)}(x) = \int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\frac{\not{k}+m_c}{k^2-m_c^2}\,. \tag{8}$$

From (6)– (8) we see that, in order to calculate the theoretical part of the correlator, the matrix elements of the non-local operators between vector ϕ meson and vacuum states are needed. We see from (6) that the contribution to the correlator comes only from the wave functions that contain an odd number of γ matrices.

Up to twist-4, the ϕ meson wave functions containing an odd number of γ matrices and appearing in our calculations are [21]

$$\begin{aligned}&\langle\phi(P,\lambda)|\bar{s}(x)\gamma_\mu s(0)|0\rangle\\ &= f_\phi m_\phi\Big[P_\mu\frac{e^\lambda x}{Px}\int_0^1\mathrm{d}u\,\mathrm{e}^{\mathrm{i}uPx}\\ &\quad\times\left(\Phi_\parallel(u,\mu^2)+\frac{m_\phi^2x^2}{16}A\left(u,\mu^2\right)\right)\\ &\quad+\left(e_\mu^\lambda-P_\mu\frac{e^\lambda x}{Px}\right)\int_0^1\mathrm{d}u\,\mathrm{e}^{\mathrm{i}uPx}g_\perp^{(v)}\left(u,\mu^2\right)\\ &\quad-\frac{1}{2}x_\mu\frac{e^\lambda x}{(Px)^2}m_\phi^2\int_0^1\mathrm{d}u\,\mathrm{e}^{\mathrm{i}uPx}C\left(u,\mu^2\right)\,,\end{aligned} \tag{9}$$

$$\begin{aligned}&\langle\phi(P,\lambda)|\bar{s}(x)\gamma_\mu\gamma_5 s(0)|0\rangle\\ &=\frac{1}{4}\left(f_\phi-\frac{2f_\phi^Tm_s}{m_\phi}\right)m_\phi\epsilon_\mu^{\ \nu\alpha\beta}e_\nu^\lambda P_\alpha x_\beta\\ &\quad\times\int_0^1\mathrm{d}u\,\mathrm{e}^{\mathrm{i}uPx}g_\perp^{(a)}\left(u,\mu^2\right)\,,\end{aligned} \tag{10}$$

$$\begin{aligned}&\langle\phi(P,\lambda)|\bar{s}(x)gG_{\mu\nu}(ux)\mathrm{i}\gamma_\alpha s(0)|0\rangle\\ &= f_\phi m_\phi p_\alpha\left(p_\nu e_{\perp\mu}^\lambda-p_\mu e_{\perp\nu}^\lambda\right)\mathcal{V}(u,px)\\ &\quad+f_\phi m_\phi^3\frac{e^\lambda x}{px}\left(p_\mu g_{\alpha\nu}^\perp-p_\nu g_{\alpha\mu}^\perp\right)\Phi(u,px)\\ &\quad+f_\phi m_\phi^3\frac{e^\lambda x}{(px)^2}p_\alpha(p_\mu x_\nu-p_\nu x_\mu)\Psi(u,px)\,,\end{aligned} \tag{11}$$

$$\begin{aligned}&\langle\phi(P,\lambda)|\bar{s}(x)g\widetilde{G}_{\mu\nu}(ux)\mathrm{i}\gamma_\alpha\gamma_5 s(0)|0\rangle\\ &= f_\phi m_\phi p_\alpha\left(p_\nu e_{\perp\mu}^\lambda-p_\mu e_{\perp\nu}^\lambda\right)\widetilde{\mathcal{V}}(u,px)\\ &\quad+f_\phi m_\phi^3\frac{e^\lambda x}{px}\left(p_\mu g_{\alpha\nu}^\perp-p_\nu g_{\alpha\mu}^\perp\right)\widetilde{\Phi}(u,px)\\ &\quad+f_\phi m_\phi^3\frac{e^\lambda x}{px}p_\alpha(p_\mu x_\nu-p_\nu x_\mu)\widetilde{\Psi}(u,px)\,.\end{aligned} \tag{12}$$

In all expressions, we have used

$$\begin{aligned}p_\mu &= P_\mu-\frac{1}{2}x_\mu\frac{m_\phi^2}{px}\,,\\ e_\mu^\lambda &= \frac{e^\lambda x}{px}\left(p_\mu-\frac{m_\phi^2}{2(px)}x_\mu\right)+e_{\perp\mu}^\lambda\,,\\ g_{\mu\nu}^\perp &= g_{\mu\nu}-\frac{1}{px}(p_\mu x_\nu+p_\nu x_\mu)\,,\end{aligned} \tag{13}$$

where $\Phi_\parallel$ is the leading twist-2 wave function, while $g_\perp^{(v)}$, $g_\perp^{(a)}$, $\mathcal{V}$ are twist-3 and all the remaining ones are twist-4 wave functions. The notation used in (11)–(14) is as follows:

$$K(u,Px) = \int\mathcal{D}\alpha\mathrm{e}^{\mathrm{i}Px(\alpha_1+u\alpha_3)}K(\alpha)\,, \tag{14}$$

where

$$\mathcal{D}\alpha = \mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)\,.$$

Inserting (7) and (8) into (6) and using the definitions of the ϕ meson wave functions, the invariant structures $\Gamma^{0,\pm,V}$ take the following form:

$$\Gamma_0 = \int\mathrm{d}u\,\frac{2f_\phi m_\phi m_c g_\perp^{(v)}(u)}{\Delta_1}, \tag{15}$$

$$\begin{aligned}\Gamma^+ &= \int\mathrm{d}u\left\{\frac{f_\phi m_\phi m_c}{\Delta_1^3}\left[m_c^2m_\phi^2A(u)+4(Pq)\Delta_1\Phi_\parallel^{(i)}(u)\right]\right.\\ &\quad-\frac{f_\phi m_\phi^3 m_c uC(u)}{\Delta_1^2}\\ &\quad\left.-\int\mathcal{D}\alpha\,\frac{f_\phi m_\phi^3 m_c}{\Delta_2^2}\left(2\Phi-2\widetilde{\Phi}+\Psi-\widetilde{\Psi}-\frac{\mathcal{V}}{2}+\frac{\widetilde{\mathcal{V}}}{2}\right)\right\},\end{aligned} \tag{16}$$

$$\begin{aligned}\Gamma^- &= \int\mathrm{d}u\left\{\frac{-f_\phi m_\phi m_c}{\Delta_1^3}\left[m_c^2m_\phi^2A(u)+4(Pq)\Delta_1\Phi_\parallel^{(i)}(u)\right]\right.\\ &\quad-\frac{f_\phi m_\phi^3 m_c(2-u)C(u)}{\Delta_1^2}\\ &\quad\left.+\int\mathcal{D}\alpha\,\frac{f_\phi m_\phi^3 m_c}{\Delta_2^2}\left(2\Phi-2\widetilde{\Phi}+\Psi-\widetilde{\Psi}-\frac{\mathcal{V}}{2}+\frac{\widetilde{\mathcal{V}}}{2}\right)\right\},\end{aligned} \tag{17}$$

$$\Gamma^V = \int \mathrm{d}u \left(1 - \frac{2f_\phi^T m_s}{f_\phi m_\phi}\right) g_\perp^{(a)} \frac{f_\phi m_\phi m_c}{\Delta_1^2}, \tag{18}$$

where

$$\Phi_\parallel^{(i)}(u) = \int_0^u \mathrm{d}v \left[\Phi_\parallel(v) - g_\perp^{(v)}(v)\right],$$
$$\Delta_1 = m_c^2 - (q + Pu)^2,$$
$$\Delta_2 = m_c^2 - [q + (\alpha_1 + u\alpha_3)P]^2,$$

Equating the expressions of the invariant structures $\Gamma^{0,\pm,V}$ coming from QCD and the phenomenological parts of the correlation function and making the Borel transformation with respect to $(P+q)^2$ in both parts, in order to suppress the contributions of higher states and continuum and also to eliminate the subtraction terms in the dispersion integral, we get the following sum rules for the $D_s \to \phi$ transition form factors:

$$A_1(q^2) = \frac{m_c + m_s}{f_{D_s} m_{D_s}^2} \frac{1}{m_{D_s} + m_\phi} \mathrm{e}^{m_{D_s}^2/M^2} \times \left\{ 2 f_\phi m_\phi m_c \int_\delta^1 \mathrm{d}u \frac{g_\perp^{(v)}(u)}{u} \mathrm{e}^{-s(u)/M^2} \right\}, \tag{19}$$

$$\begin{aligned} A_2(q^2) &= \frac{(m_c + m_s)(m_{D_s} + m_\phi)}{f_{D_s} m_{D_s}^2} \frac{2}{m_{D_s}^2 - m_\phi^2 - q^2} \mathrm{e}^{m_{D_s}^2/M^2} \\ &\times \left\{ f_\phi m_\phi m_c \left[\frac{1}{2} m_\phi^2 m_c^2 \int_\delta^1 \frac{1}{u} A(u) \frac{1}{2\left(M^2 u\right)^2} \mathrm{e}^{-s(u)/M^2} \right.\right. \\ &- \int_\delta^1 \mathrm{d}u \frac{1}{u^2} \Phi_\parallel^{(i)}(u) \mathrm{e}^{-s(u)/M^2} \\ &+ \int_\delta^1 \mathrm{d}u \frac{1}{u} \left(m_c^2 - m_\phi^2 u^2 - q^2\right) \frac{\Phi_\parallel^{(i)}(u)}{u^2 M^2} \mathrm{e}^{-s(u)/M^2} \\ &\left. - m_\phi^2 \int_\delta^1 \mathrm{d}u \frac{u C^{(i)}(u)}{u^2 M^2} \mathrm{e}^{-s(u)/M^2} \right] \\ &- f_\phi m_\phi^3 m_c \int_0^1 \mathrm{d}u\, \mathcal{D}\alpha\, \theta(s_0 - s(k)) \times \frac{1}{k^2 M^2} \\ &\times \left[2\Phi(\alpha) - 2\widetilde{\Phi}(\alpha) + \Psi(\alpha) - \widetilde{\Psi}(\alpha) \frac{\mathcal{V}}{2} + \frac{\widetilde{\mathcal{V}}}{2} \right] \\ &\left. \times \mathrm{e}^{-s(k)/M^2} \right\}. \end{aligned} \tag{20}$$

The form factor $A_3(q^2)$ can be obtained from the exact result

$$A_3(q^2) = \frac{m_{D_s} + m_\phi}{2m_\phi} A_1(q^2) - \frac{m_{D_s} - m_\phi}{2m_\phi} A_2(q^2), \tag{21}$$

and $A_0(q^2)$ can be calculated from the following sum rule:

$$\begin{aligned} A_3(q^2) - A_0(q^2) &= \frac{m_c + m_s}{f_{D_s} m_{D_s}^2} \frac{q^2}{2m_\phi} \frac{1}{m_{D_s}^2 - m_\phi^2 - q^2} \mathrm{e}^{m_{D_s}^2/M^2} \\ &\times \left\{ f_\phi m_\phi m_c \left[-\frac{1}{4} m_\phi^2 m_c^2 \int_\delta^1 \frac{1}{u} A(u) \frac{1}{2\left(M^2 u\right)^2} \mathrm{e}^{-s(u)/M^2} \right.\right. \\ &+ \int_\delta^1 \mathrm{d}u \frac{1}{u^2} \Phi_\parallel^{(i)}(u) \mathrm{e}^{-s(u)/M^2} \\ &- \int_\delta^1 \mathrm{d}u \frac{1}{u} \left(m_c^2 - m_\phi^2 u^2 - q^2\right) \frac{\Phi_\parallel^{(i)}(u)}{u^2 M^2} \mathrm{e}^{-s(u)/M^2} \\ &\left. - m_\phi^2 \int_\delta^1 \mathrm{d}u \frac{(2-u) C^{(i)}(u)}{u^2 M^2} \mathrm{e}^{-s(u)/M^2} \right] \\ &+ f_\phi m_\phi^3 m_c \int_0^1 \mathrm{d}u\, \mathcal{D}\alpha\, \theta(s_0 - s(k)) \frac{1}{k^2 M^2} \\ &\times \left[2\Phi(\alpha) - 2\widetilde{\Phi}(\alpha) + \Psi(\alpha) - \widetilde{\Psi}(\alpha) - \frac{\mathcal{V}}{2} + \frac{\widetilde{\mathcal{V}}}{2} \right] \\ &\left. \times \mathrm{e}^{-s(k)/M^2} \right\}, \end{aligned} \tag{22}$$

$$\begin{aligned} V(q^2) &= \frac{(m_{D_s} + m_\phi)(m_c + m_s)}{2 f_{D_s} m_{D_s}^2} \mathrm{e}^{m_{D_s}^2/M^2} \\ &\times \left\{ \left(1 - \frac{2 m_s f_\phi^T}{f_\phi m_\phi}\right) f_\phi m_\phi m_c \right. \\ &\left. \times \int_\delta^1 \mathrm{d}u\, g_\perp^{(a)}(u) \frac{1}{u^2 M^2} \mathrm{e}^{-s(u)/M^2} \right\}, \end{aligned} \tag{23}$$

where M^2 is the Borel parameter and

$$s(t) = \frac{m_c^2 - q^2 \bar{t} + m_\phi^2 t \bar{t}}{t},$$
$$t = \begin{cases} u, & \text{or} \\ k = \alpha_1 + u\alpha_3, \end{cases}$$
$$\bar{t} = 1 - t,$$
$$\delta = \frac{1}{2m_\phi^2} \left[\left(m_\phi^2 + q^2 - s_0\right) + \sqrt{\left(s_0 - m_\phi^2 - q^2\right)^2 - 4 m_\phi^2 \left(q^2 - m_c^2\right)} \right].$$

3 Numerical analysis

In this section we present our numerical calculation of the form factors A_1, A_2, A_0 and V. As can easily be seen from the expressions of these form factors, the main input parameters are the ϕ meson wave functions, whose explicit forms are given in [21] and that we use in our study. The values of the other input parameters appearing in the sum rules

Table 1. Comparison of our results for the form factors at $q^2 = 0$ with the results of [15, 16]

	Our result	[15]	[16]
$A_1(0)$	0.54 ± 0.09	0.55 ± 0.15	0.37 ± 0.05
$A_2(0)$	0.57 ± 0.12	0.59 ± 0.17	-0.40 ± 0.03
$A_3(0) = A(0)$	0.53 ± 0.09	0.53 ± 0.12	0.73 ± 0.06
$V(0)$	0.7 ± 0.1	1.21 ± 0.33	0.80 ± 0.09

Table 2. Parameters of the form factors given in (24), for the D_s decay in a three-parameter fit. We take the central values of the form factors for $F(0)$

	$F(0)$	a_F	b_F
A_1	0.54	1.57	0.16
A_2	0.57	4.7	5.96
A_0	0.53	0.64	2.13
V	0.64	2.81	1.34

for the form factors are $m_{D_s} = 1.9686\,\mathrm{GeV}$, $m_s(1\,\mathrm{GeV}) = (0.14 \pm 0.02)\,\mathrm{GeV}$, $m_c(1\,\mathrm{GeV}) = (1.42 \pm 0.03)\,\mathrm{GeV}$, $f_{D_s} = (0.214 \pm 0.038)\,\mathrm{GeV}$ [3], and $m_\phi = 1.02\,\mathrm{GeV}$. The leptonic decay constant of the ϕ meson, which is $f_\phi = 0.234\,\mathrm{GeV}$, is extracted from the experimental result of the $\phi \to \ell^+ \ell^-$ decay [22]. As has already been noted, in order to avoid contributions of the scalar mesons, the threshold parameter s_0 in the dispersion integral must be chosen near the squared mass of the lowest scalar meson. Therefore, in our further numerical analysis, we use $s_0 = 5.0\,\mathrm{GeV}^2$ and $s_0 = 5.3\,\mathrm{GeV}^2$, both of which are slightly lower than the mass square of the scalar meson.

With the above-mentioned input parameters, we now proceed by carrying out our numerical analysis. The first step, according to the sum rule philosophy, is to look for a working region of the auxiliary Borel parameter M^2, where numerical results should be stable for a given threshold s_0. The lower limit of M^2 is determined by the condition that the terms M^{-2n} $(n > 1)$ remain subdominant. The upper bound of M^2 is determined by requiring that the continuum and higher state contributions constitute a maximum 30% of the total result. Our numerical analysis shows that both requirements are satisfied in the region $3\,\mathrm{GeV}^2 \leq M^2 \leq 4.5\,\mathrm{GeV}^2$. We should note that LCQSR for the form factors are reliable in the region $q^2 \lesssim 0.4\,\mathrm{GeV}^2$. Moreover, we analyze the M^2 dependencies of the form factors A_1, A_2, A_0 and V at $q^2 = 0\,\mathrm{GeV}^2$ and $q^2 = 0.2\,\mathrm{GeV}^2$ for two different values of the continuum threshold, namely $s_0 = 5.0$ and $s_0 = 5.3\,\mathrm{GeV}^2$. Our analysis shows that the form factors are practically independent of the Borel mass when M^2 varies between $3\,\mathrm{GeV}^2$ and $4\,\mathrm{GeV}^2$. The variation of the form factors in relation to the continuum threshold is also very weak. The results for all form factors change about 5% at $q^2 = 0$. Our final results for the form factors at $q^2 = 0$ and $s_0 = 5.3\,\mathrm{GeV}^2$ are presented in Table 1. For a comparison, the results of [15, 16] on the same form factors are also listed in this table.

From this table we see that, except for the value of $V(0)$, our results are close to the predictions of [15], while they differ from those given in [16]. Moreover, the sign of $A_2(0)$ in our case is different compared to that obtained in [16].

A few words about the magnitude of SU(3) violation in the form factors of the $D_s \to \phi$ and $D \to \rho$ transitions are in order. The form factors of the $D \to \rho$ transition are calculated in [23], having the values $A_1(0) \simeq 0.55$, $A_2(0) \simeq 0.6$ and $V(0) \simeq 0.9$ at $q^2 = 0$. When we compare these results with ours, we observe that the violation due to SU(3) is quite small for $A_1(0)$ and $A_2(0)$, while it is about 20% smaller for $V(0)$ (for central values).

It should be noted that in the region $q^2 \geq 0.4\,\mathrm{GeV}^2$ the applicability of the light cone QCD sum rule is questionable. In order to extend our results to the full physical region, we look for a parameterization of the form factors in such a way that in the region $0 \leq q^2 \leq 0.4\,\mathrm{GeV}^2$, the above-mentioned parameterization coincides with the light cone QCD sum rules prediction. The most convenient parameterization of the q^2 dependence of the form factors is given in terms of three parameters in the following form:

$$F_i(q^2) = \frac{F_i(0)}{1 - a_{F_i}\left(q^2/m_{D_s}^2\right) + b_{F_i}\left(q^2/m_{D_s}^2\right)^2} \,. \quad (24)$$

The values of the parameters $F_i(0)$, a_{F_i} and b_{F_i} are listed in Table 2.

We proceed by discussing the uncertainties related to the input parameters and wave functions. We note first that the radiative corrections to the leading twist-2 function, which is calculated in [20], is about $\sim 10\%$. As has already been noted, the results depend weakly on the continuum threshold s_0 and the Borel parameter M^2, and the uncertainty due to these parameters is about 5%–7% in the working region of M^2. Moreover, the results are also quite weakly dependent on the vector meson decay constant f_ϕ and f_ϕ^T, which results in an uncertainty of about 5%. The additional uncertainties coming from the Gegenbauer moments are about $\sim 10\%$. Summing all these above-mentioned errors, the overall uncertainty in the values of the form factors is of the order of 17%.

A few words about the parameterization of the form factors, given in (24), are in order. In principle, one can use the meson pole-dominance approximation in the parameterization of the form factors, which works good at large $q^2 \sim (m_{D_s} - m_\phi)^2$ and with the form factors being expressed as

$$F_i(q^2) = \frac{F_i(0)}{1 - q^2/m_i^2} \,.$$

However, it is not obvious at all that the meson-dominance model gives reliable results at low q^2. On the other hand, it is well known that the QCD sum rules prediction works very well at low q^2. So, one can use both parameterizations in the following way: match both representations of the form factors ((24) and the pole form) and treat $F_i(0)$ as the fit parameter, and further require that at some value $q^2 = q_0^2$ to be fitted, the values of $F_i(q^2 = q_0^2)$ and their derivatives

Table 3. Comparison of our results for r_1 and r_2 with the experimental results and the 3-point sum rule

	r_1	r_2
Focus [24]	1.549 ± 0.250	$0.713 \pm 0.202 \pm 0.266$
E791 [14]	$2.27 \pm 0.35 \pm 0.22$	$1.57 \pm 0.25 \pm 0.19$
CLEO [13]	$0.9 \pm 0.6 \pm 0.3$	$1.4 \pm 0.5 \pm 0.3$
E687 [12]	$1.8 \pm 0.9 \pm 0.2$	$1.1 \pm 0.8 \pm 0.1$
E653 [11]	$2.3^{+1.1}_{-0.9} \pm 0.4$	$2.1^{+0.6}_{-0.5} \pm 0.2$
Average	1.92 ± 0.32	1.60 ± 0.24
3PSR [15]	2.20 ± 0.85	1.16 ± 0.46
3PSR [16]	2.16 ± 0.38	-1.08 ± 0.17
Our results	1.19 ± 0.23	1.06 ± 0.24

are equal in both parameterizations. After carrying out this procedure, one can use both parameterizations, i.e., one can use (24) for $q^2 < q_0^2$ and the pole form for $q^2 > q_0^2$.

In the experiments, the ratios

$$r_1 = \frac{V(0)}{A_1(0)} \quad \text{and} \quad r_2 = \frac{A_2(0)}{A_1(0)}$$

are measured. In the present work, within the framework of the light cone QCD sum rules, we get $r_1 = 1.19 \pm 0.23$ and $r_2 = 1.06 \pm 0.24$. In Table 3, we present a comparison of our results with the existing experimental data and 3-point sum rule (3PSR).

Using the parameterization of the $D_s \to \phi$ transition in terms of the form factors $A_1, A_2, V, A_3 - A_0$, the differential decay width as a function of q^2 in terms of the helicity amplitudes can be written as

$$\begin{aligned} \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} &= \frac{G_{\mathrm{F}}^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \lambda^{1/2}\left(m_{D_s}^2, m_\phi^2, q^2\right) q^2 \left[H_0^2 + H_+^2 + H_-^2\right] \\ &\equiv \frac{\mathrm{d}\Gamma_{\mathrm{L}}}{\mathrm{d}q^2} + \frac{\mathrm{d}\Gamma_+}{\mathrm{d}q^2} + \frac{\mathrm{d}\Gamma_-}{\mathrm{d}q^2}, \end{aligned} \tag{25}$$

where the indices in $\mathrm{d}\Gamma_i/\mathrm{d}q^2$ and H_i denote the polarization of the ϕ meson, $\lambda\left(m_{D_s}^2, m_\phi^2, q^2\right) = \left(m_{D_s}^2 + m_\phi^2 - q^2\right)^2 - 4m_{D_s}^2 m_\phi^2$, and

$$H_\pm = (m_{D_s} + m_\phi) A_1(q^2) \mp \frac{\lambda^{1/2}\left(m_{D_s}^2, m_\phi^2, q^2\right)}{m_{D_s} + m_\phi} V(q^2), \tag{26}$$

$$\begin{aligned} H_0 = &\frac{1}{2m_\phi\sqrt{q^2}} \\ &\times \left[\left(m_{D_s}^2 - m_\phi^2 - q^2\right)(m_{D_s} + m_\phi) A_1(q^2)\right. \\ &\left. - \frac{\lambda\left(m_{D_s}^2, m_\phi^2, q^2\right)}{m_{D_s} + m_\phi} A_2(q^2)\right]. \end{aligned} \tag{27}$$

The differential decay rate when the final state ϕ meson is transversally polarized is determined to be

$$\frac{\mathrm{d}\Gamma_{\mathrm{T}}}{\mathrm{d}q^2} = \frac{\mathrm{d}\Gamma_+}{\mathrm{d}q^2} + \frac{\mathrm{d}\Gamma_-}{\mathrm{d}q^2}. \tag{28}$$

Integrating the differential decay widths over q^2 in the region from $q^2 = 0$ to $(m_{D_s} - m_\phi)^2$, we obtain

$$\Gamma_{\mathrm{L}} = \left(1.31^{-0.13}_{+0.10}\right) \times 10^{-14}\,\mathrm{GeV},$$

$$\Gamma_{\mathrm{T}} = \left(1.57^{-0.28}_{+0.27}\right) \times 10^{-14}\,\mathrm{GeV},$$

and for their ratio we get

$$\frac{\Gamma_{\mathrm{L}}}{\Gamma_{\mathrm{T}}} = (0.8 \pm 0.1),$$

which is in good agreement with the existing experimental data,

$$\left(\frac{\Gamma_{\mathrm{L}}}{\Gamma_{\mathrm{T}}}\right)_{\mathrm{exp}} = 0.72 \pm 0.16 \quad [22].$$

Using the value of the total decay width $\Gamma_{D_s} = 1.34 \times 10^{-12}$ GeV [22] of the D_s meson, we get the following result for the branching ratio of the $D_s \to \phi\bar{\ell}\nu$ decay:

$$\mathcal{B}\left(D_s \to \phi\bar{\ell}\nu\right) = \left(2.15^{-0.31}_{+0.27}\right)\%,$$

which is consistent with the experimental result

$$\mathcal{B}\left(D_s \to \phi\bar{\ell}\nu\right)_{\mathrm{exp}} = (2.0 \pm 0.5)\%.$$

In conclusion, we calculate the form factors for the $D_s \to \phi$ transition, in the framework of the light cone QCD sum rules. We compare our results for the form factors with the existing calculations based on 3-point sum rules. Following this analysis, we then estimate the ratios of these form factors and compare them with the current experimental data, as well as with the existing theoretical calculations. Finally, we study the ratio $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{T}}$ of the decay widths when the ϕ meson is longitudinally and transversally polarized, and the branching ratio. Our calculations on the abovementioned quantities confirm that they are consistent with the existing experimental data.

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